Global Trade, Tariff Uncertainty and the U.S. Dollar*

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Abstract

We analyze how tariff uncertainty affects exchange rates, motivated by the U.S. dollar's depreciation after the 2025 tariff announcements. Standard macro-trade models predict that unilateral tariffs appreciate the implementing country's currency, but we show this result can be overturned by policy uncertainty. We build a two-country general equilibrium model with risk-averse agents and segmented financial markets, where tariff volatility enters uncovered interest parity through a risk-premium wedge. Higher tariff uncertainty increases precautionary savings and risk premia, leading to immediate currency depreciation even as tariffs rise. Quantitatively, the model replicates the size and timing of the observed dollar depreciation episode dynamics.

JEL Codes: E2, E3, E6, F1, F4
Keywords: Tariffs, tariff uncertainty, exchange rate volatility, UIP.

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Standard models predict appreciation of the currency of the tariff-imposing country in the case of unilateral tariffs; in reality, the US dollar has depreciated substantially since the first quarter of 2025, when unilateral tariffs were imposed.[1] Figure 1a depicts changes in the nominal Broad U.S. Dollar Index since January 2024 with dashed vertical lines marking the U.S. election, the inauguration, and Liberation Day, respectively. As the figure shows, the US dollar began to depreciate following the inauguration (January 20 2025), losing 1.52% of its value until April 1, 2025 (immediately prior to Liberation Day) with an additional 1.04% between April 1 and April 10, 2025.[2] In this paper, we extend Kalemli-Özcan, Soylu and Yildirim (2025) (henceforth, KSY) to explain this unexpected depreciation. We find that while changes in tariff levels are appreciationary, increases in the volatility of tariffs can be depreciationary when households and financial intermediaries are sensitive to risk.

Such sensitivity to risk can be picked up by the UIP premium. Figure 1b shows that the UIP premium indeed widened in 1Q2025, relative to the 4Q2024, as the US dollar depreciated. A standard second-order log approximation to the UIP condition in the finance literature is: $log(1+i_{H,t}) - log(1+i_{F,t}) = \mathbb{E}_t[\log(\mathcal{E}_{t+1}) - \log(\mathcal{E}_t)] + \frac{1}{2}\mathrm{Var}_t(\log(\mathcal{E}_{t+1}) - \log(\mathcal{E}_t))$, where $\mathcal{E}_t$ denotes the nominal exchange rate, defined as the price of foreign currency in units of home currency and $i_{H,t}$ ($i_{F,t}$) is the nominal interest rate in the Home (Foreign) country. The added volatility term in general is regarded as small noise.

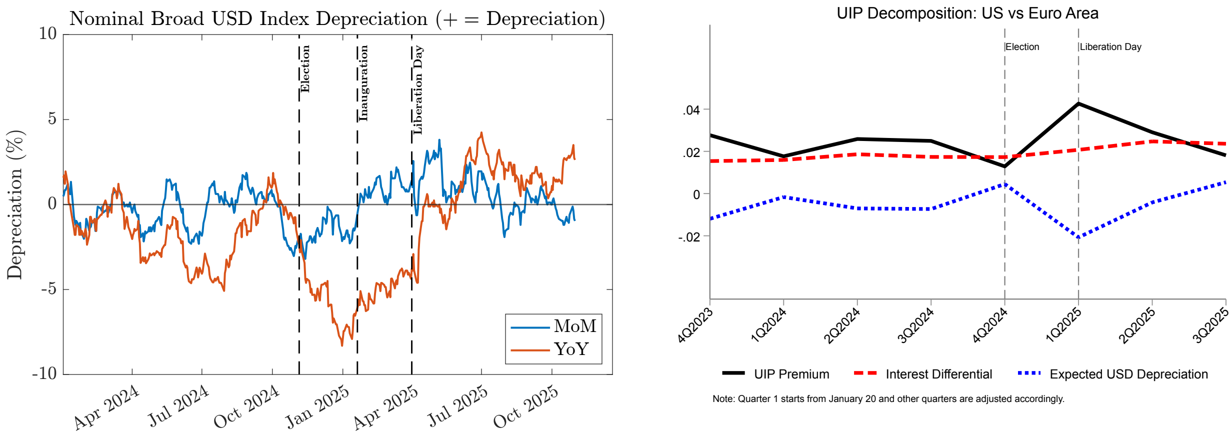
We write, based on our model, the linearized uncovered interest parity (UIP) condition: $\hat{i}_{H,t} - \hat{i}_{F,t} = \mathbb{E}_t\left[\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t\right] + \rho_t$, where ρ_t is the time-varying currency risk or the UIP premium and hat variables denote deviation from the steady state. When the Home country imposes import tariffs with uncertainty, that is, when announced tariff rates may deviate from ultimately implemented rates, this time varying currency risk can become quite large.[3]

[1] While we focus on the nominal exchange rate in this paper, the dollar depreciated both in real and nominal terms. The mechanism proposed in this paper explains both real and nominal depreciation.

[2] We use this period to cover Liberation Day (April 2, 2025), subsequent escalation and the pause on April 9.

[3] This is because, in our setup, ρ_t is not a small Jensen's inequality correction originating from a second-order approximation (e.g. $\rho_t = \frac{1}{2}\mathrm{Var}(\log(\mathcal{E}_{t+1}) - \log(\mathcal{E}_t))$); instead, it is micro-founded through the intermediaries' mean–variance optimization problem and is nonzero even prior to approximation.

Figure 1. US Dollar Depreciation Measures



(a) Broad US Dollar Index **(b)** UIP Decomposition

Note: The left panel shows month-on-month and year-on-year changes in the broad US dollar index, whereby positive values indicate depreciation of the US dollar and negative values indicate appreciation. The figure highlights the partial reversal of the strong 2024 appreciation in 2025. The right panel presents a decomposition of the UIP condition into three components: the interest rate differential, $\hat{i}_{H,t} - \hat{i}_{F,t}$*; expected exchange rate depreciation,* $\mathbb{E}_t\left[\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t\right]$*; and the UIP premium,* ρ_t*. As the UIP premium rose, the USD depreciated. Because the depreciation occurred contemporaneously, agents expected an appreciation of the dollar, while the interest rate differential has been relatively stable. To align the data with the timing of tariff policy changes, we assume the first quarter of 2025 starts on January 20, 2025 with the inauguration of the president to capture both announcements around Liberation Day and earlier tariff rate increases in the same quarter; all other quarters are adjusted accordingly. We follow the methodology of Kalemli-Özcan (2019) to construct the UIP premium. The nominal broad US dollar index is sourced from FRED, and exchange rate expectations are taken from Consensus surveys.*

Interpreting this as perceptions of risk widening around tariff announcements, we extend KSY to explain the dollar depreciation. KSY analyze the macroeconomic effects of trade distortions within a global dynamic general equilibrium framework featuring multi-sector, multi-country production networks with full input–output linkages and nominal rigidities under incomplete markets. KSY show that increases in tariff rates can generate global inflation and lower output, accompanied by an appreciation of the home currency. In contrast, tariff threats—announcements that never get implemented—generate deflationary pressures combined with a modest depreciation of the dollar after tariff announcements are reversed.

In this paper, we simplify features of KSY and introduce two additional elements: risk-sensitive financial intermediaries and CARA utility. We assume financial intermediaries solve a mean–variance portfolio problem over Home and Foreign bonds in segmented financial markets.[4] This extension allows us to analytically characterize the conditions under which home currency depreciation can arise following announcements of home tariffs that are uncertain in nature—that is, when agents do not know the future level of tariffs and the range of possible future tariff outcomes widens today. Solving the intermediaries' problem shows that ρ_t depends positively on the conditional variance of exchange rate movements, scaled by a parameter χ (i.e., $\rho_t = \chi \text{Var}_t(\hat{\mathcal{E}}_{t+1})$), where χ need not be small.[5] In our framework, this conditional variance is directly linked to tariff uncertainty, so increases in tariff volatility widen the UIP wedge. At the same time, greater volatility in expected tariffs acts as an Euler equation shock for risk-sensitive households (via $\text{Var}(\hat{C}_{t+1})$), inducing precautionary savings. As a result, heightened uncertainty about future tariff policy can generate contemporaneous currency depreciation via these two channels even as tariff levels rise. This modeling approach aligns well with reality, as the Liberation Day announcements generated shocks not only to tariff levels but also to tariff volatility, with both trade policy uncertainty and broader economic policy uncertainty indices peaking around the announcement, as shown in the Supplementary Appendix.

1 Model

We develop a simplified two-country, one-good version of KSY to illustrate the mechanism through which tariff uncertainty can generate a depreciation of the dollar. Relative to KSY, we feature an endowment economy and monetary policy is simplified by fixing the aggregate price level (CPI). In addition, we assume symmetry and set both the intra- and inter-temporal elasticities of substitution to one. We further assume that only home households exhibit CARA utility. Finally, we introduce financial intermediaries that solve a mean–variance

[4] CARA utility and segmented financial markets are absent in KSY, which features incomplete financial markets.

[5] In our stylized setup, we assume investors are sensitive to the price of risk given by the variance term, but not to the quantity of risk. In SA, we present the more general case in which the level of outstanding debt enters into ρ_t.

portfolio optimization problem. Under these assumptions, the five-equation system in KSY collapses to a tractable set of equilibrium conditions that capture the key mechanisms of interest. Full derivations are provided in SA.

When a shock occurs, we track changes in variables as percent deviations from their steady-state values, denoted by a caret (ˆ). Lack of time notation for a variable denotes steady-state level (e.g., R_H (R_F) is the steady-state level of the gross nominal interest rate for the Home (Foreign) country). Steady-state unconditional moments (e.g., variances) are evaluated at the ergodic distribution, i.e., under non-zero volatility. $\hat{p}_{H,t}$ ($\hat{p}_{F,t}$) denotes the price of Home (Foreign) goods produced and consumed domestically (abroad). $\hat{\mathcal{E}}_t$ is the nominal exchange rate and real exchange rate (since aggregate price levels are fixed) where an increase corresponds to a depreciation of the home currency. $\hat{V}_{H,t}$ is the net debt position of the Home country, inclusive of interest payments. $\hat{C}_{H,t}$ and $\hat{C}_{F,t}$ denote consumption by Home and Foreign households. $(1-\gamma)$ captures home bias in consumption.

Definition 1. An approximated equilibrium comprises 8 sequences $\{\hat{p}_{H,t}, \hat{p}_{F,t}, \hat{\mathcal{E}}_t, \hat{i}_{H,t}, \hat{i}_{F,t}, \hat{V}_{H,t}, \hat{C}_{H,t}, \hat{C}_{F,t}\}_{t=0}^{\infty}$ such that, given exogenous variables $\{\hat{\tau}_t, \sigma_t^2\}_{t=0}^{\infty}$, the equations (1)-(8) hold:

Euler equations with Home country exhibiting CARA utility:

$$(\mathbb{E}_t\hat{C}_{H,t+1} - \hat{C}_{H,t}) = \hat{i}_{H,t} + \underbrace{\frac{1}{2}\mathrm{Var}_t(\hat{C}_{t+1})}_{\eta\sigma_t^2}, \qquad (\mathbb{E}_t\hat{C}_{F,t+1} - \hat{C}_{F,t}) = \hat{i}_{F,t}. \tag{1-2}$$

Aggregate price level with policy stabilizing aggregate price levels in both countries:

$$0 = (1-\gamma)\,\hat{p}_{H,t} + \gamma\,(\hat{\mathcal{E}}_t + \hat{p}_{F,t} + \hat{\tau}_t), \qquad 0 = (1-\gamma)\,\hat{p}_{F,t} + \gamma(\hat{p}_{H,t} - \hat{\mathcal{E}}_t) \tag{3-4}$$

UIP condition holds with a wedge that depends on the variance of the exchange rate:

$$\hat{i}_{H,t} - \hat{i}_{F,t} = \mathbb{E}_t[\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t] + \underbrace{\chi \text{Var}_t(\hat{\mathcal{E}}_{t+1})}_{\kappa\sigma_t^2} \tag{5}$$

Goods market clears for each country in both periods:

$$(1-\gamma)\left(\hat{C}_{H,t} - \hat{p}_{H,t}\right) + \gamma\left(\hat{C}_{F,t} + \hat{\mathcal{E}}_t - \hat{p}_{H,t}\right) = 0 \tag{6}$$

$$\gamma\left(\hat{C}_{H,t} - \hat{\mathcal{E}}_t - \hat{p}_{F,t} - \hat{\tau}_t\right) + (1-\gamma)\left(\hat{C}_{F,t} - \hat{p}_{F,t}\right) = 0 \tag{7}$$

Balance of payments equation is given by:

$$R_H^{-1}\hat{V}_{H,t} = \hat{V}_{H,t-1} - \gamma\left[(\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - (\hat{C}_{H,t} - \hat{\tau}_t)\right] \tag{8}$$

The model features two shock variables: $\hat{\tau}_t$ and $\hat{\sigma}_t^2$. The first is a one-time shock to the level of tariffs, expressed as a deviation from the steady state, with $\hat{\tau}_t \sim \mathcal{N}(0, \sigma_{t-1}^2)$. We allow the variance of $\hat{\tau}_t$ to vary exogenously over time, which constitutes the second shock. Importantly, our timing convention assumes that the variance of shocks at time $t+1$ is known and determined at time t. This structure captures tariff uncertainty: the range of possible future tariff realizations widens today. We consider one-time shocks to both the level and the variance of tariffs.

Our model is largely linear, with the exception of terms involving variances. We therefore perform a second-order approximation and simplify cross terms that are quantitatively negligible. As shown in SA, our setup and timing convention allows us to approximate the variance of the response of endogenous variables. We solve the model to obtain the policy functions for $\hat{C}_{H,t}$ and $\hat{\mathcal{E}}_t$ as a function of $\hat{\tau}_t$. Since $\hat{\tau}_t$ is the only source of uncertainty, the conditional variances can be approximated as $\text{Var}_t(\hat{C}_{H,t+1}) \approx \eta\,\sigma_t^2$ and $\text{Var}_t(\hat{\mathcal{E}}_{t+1}) \approx \kappa\,\sigma_t^2$, where η and κ are given by the squares of the coefficients on $\hat{\tau}_{t+1}$ in the respective policy functions. This yields a system that is linear in the shock variables. We solve the model using the method of undetermined coefficients. We find that tariff level shocks are appreciationary (captured by the first term below), whereas shocks to tariff volatility are depreciationary (the second term below).

$$\hat{\mathcal{E}}_t = \underbrace{\left(\left(R_H^{-1} - \tfrac{1}{2}\right)(1-2\gamma)^2 - \tfrac{1}{2}\right)}_{<0} \hat{\tau}_t + \underbrace{R_H^{-1}(1-2\gamma)^2(\eta+\kappa)}_{>0} + \underbrace{\frac{\left(1 - R_H^{-1}\right)(1-2\gamma)^2}{\gamma}}_{>0} \hat{V}_{H,t-1}$$

Higher tariff uncertainty induces precautionary saving behavior, as it effectively serves as an Euler equation shock (e.g., similar to a patience shock). This increase in precautionary savings reduces current demand and leads to a depreciation of the home currency, because under home bias, each country consumes a larger share of its own goods and a decline in domestic demand reduces relative demand for the Home country's goods. Simultaneously, higher policy volatility induces financial intermediaries to demand a higher risk premium, (e.g., similar to a country risk shock or financial intermediation shock that widens the UIP premium). All else equal, this additionally generates depreciation pressure while reducing aggregate consumption in the Home country relative to the Foreign country.

When both shocks (tariff levels and tariff volatility) are present, the overall effect on the exchange rate is ambiguous. When the sensitivity to tariff volatility in the Euler equation, captured by η, the sensitivity of investors to tariff-related volatility κ, and the underlying volatility of tariffs captured by σ_t are sufficiently large, it is possible for tariff volatility shocks to yield depreciation even at the same time as a tariff hike. Thus, whether appreciationary forces from tariff levels or depreciationary forces from tariff volatility dominate is a quantitative question, which we turn to next.

2 Data and Construction of Shocks

We obtain all tariff data from WTO-IMF Tariff Tracker.[6] The final implemented tariffs are obtained as of October 17, 2025 and differ substantially from those announced on Liberation Day. Subsequent changes in implemented tariffs after this date are negligible.

To compute the standard deviation of tariffs, we compile all tariff-related events, including announced and threatened tariffs, from the Trade Compliance Resource Hub.[7] We then compute the standard deviation across all distinct tariff rates reported in this dataset, which yields a value of 43.9%.[8] This measure captures the extent of policy uncertainty, including erratic announcements, such as the proposed 125% tariffs on China, which we believe played an important role in amplifying tariff uncertainty. We additionally construct a time series of tariff volatility from these data. For each date, we compute the standard deviation of all tariffs that have been announced or threatened up to that point in time. Tariff volatility is 72% during the first quarter of 2025, declines to 60% by the end of the second quarter, and falls further to 49% by the end of the third quarter.[9]

3 Quantitative Exercise

We feed two shocks into the model: a level shock to tariffs and a volatility shock. Specifically, the level of tariffs increases by 18.9 percentage points, while the variance of tariffs rises by 72.1 percentage points relative to their steady-state values.[10] Both shocks are introduced

[6] https://ttd.wto.org/en/analysis/tariff-actions, last accessed on October 17, 2025.

[7] https://www.tradecomplianceresourcehub.com/2025/12/11/trump-2-0-tariff-tracker/, last accessed on December 27, 2025.

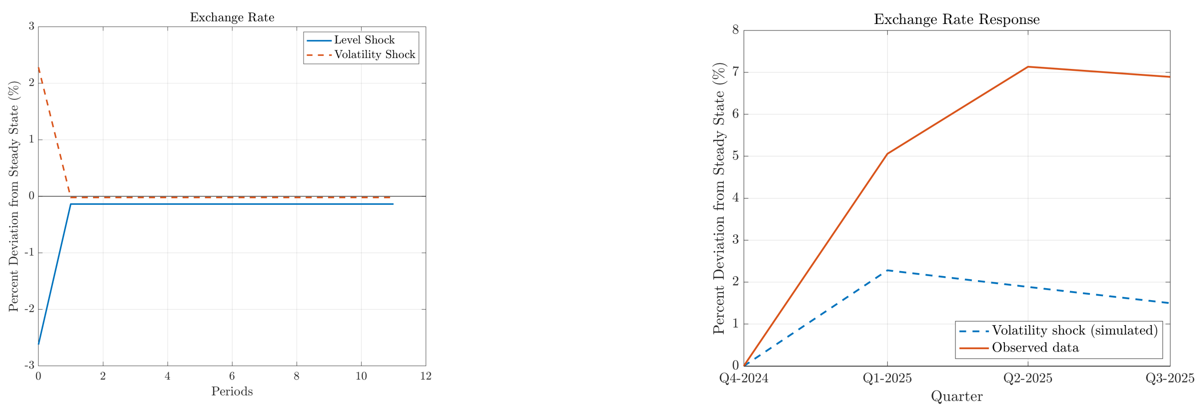
[8] We also calculate the standard deviation of all implemented tariff rates (as reported in Table A.1 of SA), which yields a value of 5.4%. In another approach, for each country–HS6 product pair with nonzero import values in 2024, we compute the standard deviation of implemented tariffs over time which yields the standard deviation of 8% at the beginning of the year and 34.9% around Liberation Day. We used these measures for robustness purposes.

[9] Our tariff volatility measure follows a similar path to the Trade Policy Uncertainty indices shown in SA.

[10] The 18.9 percentage point tariff level shock is calculated as the difference between April 9 and January 1 effective tariff rates.

as one-time innovations. As noted above, the analytical solution yields exact coefficients for κ and η dependent on model primitives like γ. Based on KSY, we calibrate the home-bias parameter to $\gamma = 0.0708$, equal to the foreign expenditure share in U.S. final consumption. As detailed in SA, the UIP wedge contains a risk sensitivity parameter, χ; we calibrate this to match the deviation of the UIP wedge from the last quarter of 2024, which we treat as the steady state as shown in Figure 1b. [11]

Figure 2. Exchange Rate Responses to Tariff Level and Volatility Shocks



(a) Exchange Rate Responses to Level and Volatility Shocks

(b) Comparing Model Response to Data: Evolving Tariff Volatility

Note: Figure 2 shows the model-implied responses to shocks in $\hat{\tau}_t$ *and* σ_t^2*. Panel (b) additionally compares the model-implied dynamics with the observed exchange rate response under evolving tariff volatility.*

Consistent with the model's predictions, Figure 2a shows that tariff level shocks are appreciationary, whereas increases in tariff volatility are depreciationary. In particular, the level shock generates a 2.6% appreciation of the exchange rate, while the volatility shock leads to a 2.3% depreciation. These numbers are highly sensitive to parametrization and assumptions; the model here is a parsimonious one.[12] With that caveat, this exercise shows that there are two competing pressures from the introduction of tariffs in the first quarter of 2025. One that raised the level of tariffs, and thereby created appreciationary pressure and

[11] Notably, exchange rate volatility varies considerably across time horizons and currency measures. The volatility of the dollar–euro exchange rate (broad dollar index) between April 1 and (i) June 30 is 36% (16%); (ii) October 1 is 27% (11%); (iii) December 1 is 23% (10%).

[12] For example, persistence (and perceived persistence) of the shocks matter significantly, as explored in KSY. If agents expect tariffs not to persist, the appreciationary effect of tariff levels can be dampened. Conversely, if tariff volatility is persistent rather than transitory, its depreciationary effect can be amplified.

another that widened the range of possible future tariffs leading to depreciationary pressure. These pressures are large enough that under different parametrizations and with a more detailed model, one can match more closely the path of the observed exchange rate. With differing elasticities of substitution and parameter asymmetry across countries, as in KSY, the appreciationary impact of tariffs can be muted to the point that the volatility shock's depreciationary impact sufficiently exceeds the appreciationary impact of tariffs.

Our baseline model features one-time volatility shocks with constant loadings. We next consider the impulse responses resulting from a series of volatility shocks that reveal themselves in successive periods. We use time-varying tariff volatility series and feed this series into the model as a sequence of one-time volatility shock, in the spirit of the tariff-threat shocks studied by KSY. We construct a cumulative IRF from these successive shocks and compare the model-implied dynamics to the percent deviation of the quarterly Nominal Broad U.S. Dollar Index from its 4Q2024 level. Figure 2b depicts the path of the exchange rate following these successive volatility shocks. Nearly half of the exchange rate response on impact in the first quarter of 2025 can be explained with the volatility shock.[13]

4 Conclusion

We study how tariff uncertainty affects exchange rate. While standard models predict that tariffs lead to currency appreciation, we show that this result can be overturned when tariff policy uncertainty increases. Uncertainty reduces demand for goods and dollars and generates a risk-premium wedge that weakens the currency of the tariff-imposing country. Our quantitative results can account for a sizable share of the observed dollar depreciation.

What do these results imply for "exorbitant privilege"? USD dominance rests on its functional advantages, network effects, and its perceived safety as a store of value. These

[13] The broad dollar index (and against euro) depreciated between April 1 and (i) April 10 around 1-2%, (ii) June 30 by 5% (9%), (iii) October 1 by 4.8% (8.5%) (iv) December 1 by 4% (7.5%).

can come in question due to major policy errors, including uncertain trade policies, that may accelerate an erosion of confidence already underway (Rogoff, 2025).

Trade policy uncertainty is not merely a second-order feature. Incorporating uncertainty (Akıncı, Kalemli-Özcan and Queralto, 2022) into macro models is essential for understanding exchange rate movements during periods of geopolitical instability.

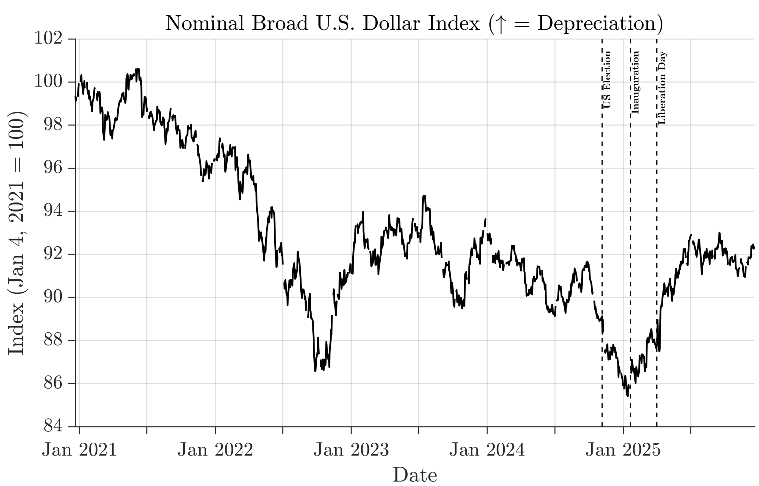
Supplemental Appendix

Global Trade, Tariff Uncertainty and the U.S. Dollar

Şebnem Kalemli-Özcan, Can Soylu and Muhammed A. Yıldırım

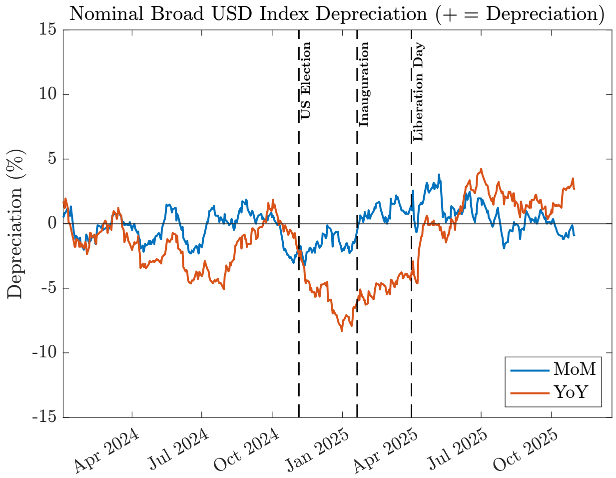
A Additional Figures and Table

Figure A.1. Nominal Broad US Dollar Index

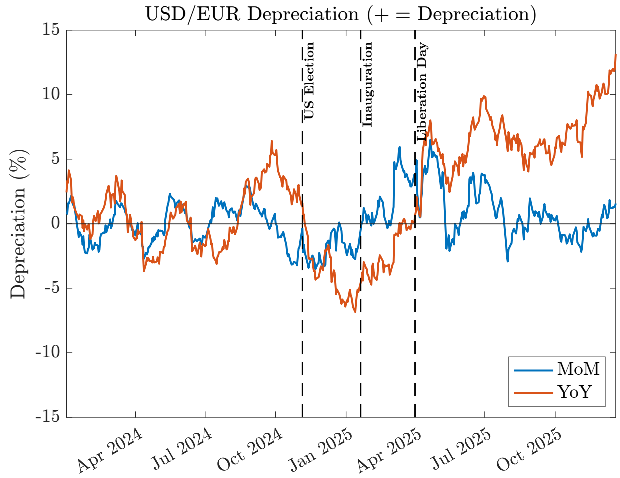


Note: The broad U.S. Dollar index since January 1, 2021. Source: FRED.

Figure A.2. US Dollar Depreciation Measures



(a) Broad US Dollar Index



(b) USD–EUR Exchange Rate

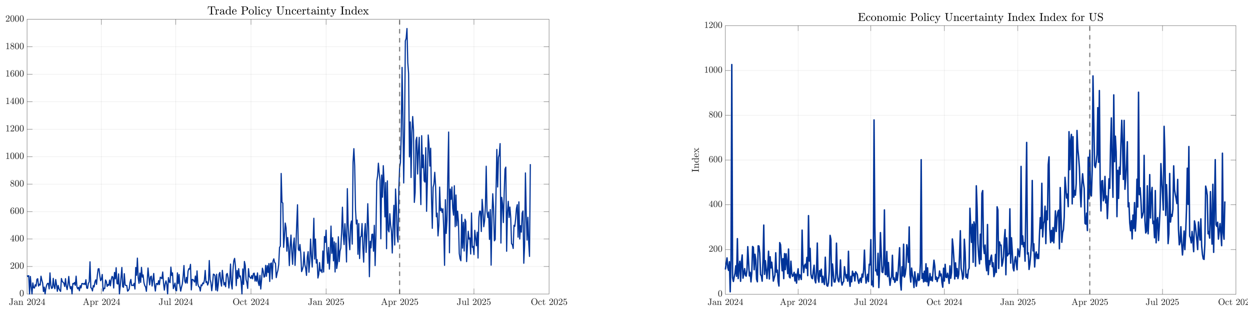
Note: Positive values indicate depreciation of the US dollar and negative values indicate appreciation. The left panel shows month-on-month and year-on-year changes in the broad US dollar index, highlighting the partial reversal of the strong 2024 appreciation in 2025. The right panel shows USD–EUR depreciation, where the depreciation pattern is more pronounced. Source: FRED.

Table A.1. U.S. Dollar Depreciation/Appreciation

	Broad US Dollar Index		**USD–EUR**	
Date	MoM (30d, %)	YoY (365d, %)	MoM (30d, %)	YoY (365d, %)
2025-04-01	1.4	-3.8	3.8	0.6
2025-04-02	1.0	-3.9	3.5	0.9
2025-04-03	2.6	-2.9	4.9	2.1
2025-04-04	0.5	-3.9	2.3	1.4
2025-04-07	-0.6	-4.5	0.5	0.7
2025-04-08	-0.6	-4.6	0.5	0.5
2025-04-09	-0.3	-4.6	1.9	1.7
2025-04-10	0.8	-2.7	2.4	4.2

Note: Positive values indicate depreciation of the US dollar; negative values indicate appreciation. Source: FRED.

Figure A.3. Measures of Policy Uncertainty



(a) Trade Policy Uncertainty

(b) U.S. Economic Policy Uncertainty

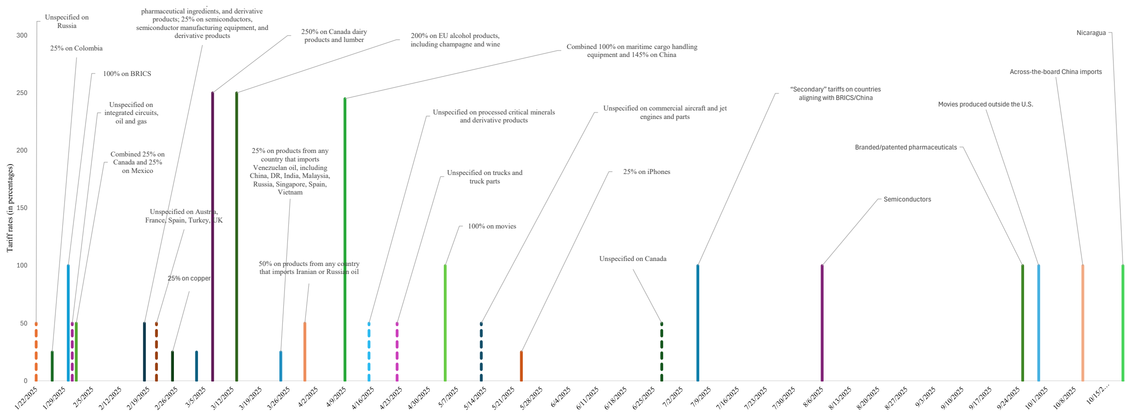
Note: *Trade and economic policy uncertainty indices of Baker, Bloom and Davis (2016) and Caldara et al. (2020) from January 2024 to October 2025. Both series rise sharply around the Liberation Day tariff announcements and remain elevated afterward. Although some uncertainty dissipates, each measure stays above its level from the prior year.*

Table A.2. U.S. Weighted Average Implemented Tariff Rate

Date	Tariff (%)	Date	Tariff (%)
1-Jan-25	2.5	23-Jun-25	15.6
4-Feb-25	3.9	30-Jun-25	15.6
4-Mar-25	11.7	1-Aug-25	15.6
7-Mar-25	6.3	6-Aug-25	15.8
12-Mar-25	7.4	7-Aug-25	17.0
3-Apr-25	8.5	27-Aug-25	17.4
5-Apr-25	12.2	1-Sep-25	17.4
9-Apr-25	21.4	8-Sep-25	17.3
3-May-25	23.0	16-Sep-25	17.2
14-May-25	13.8	1-Oct-25	17.2
4-Jun-25	15.5		

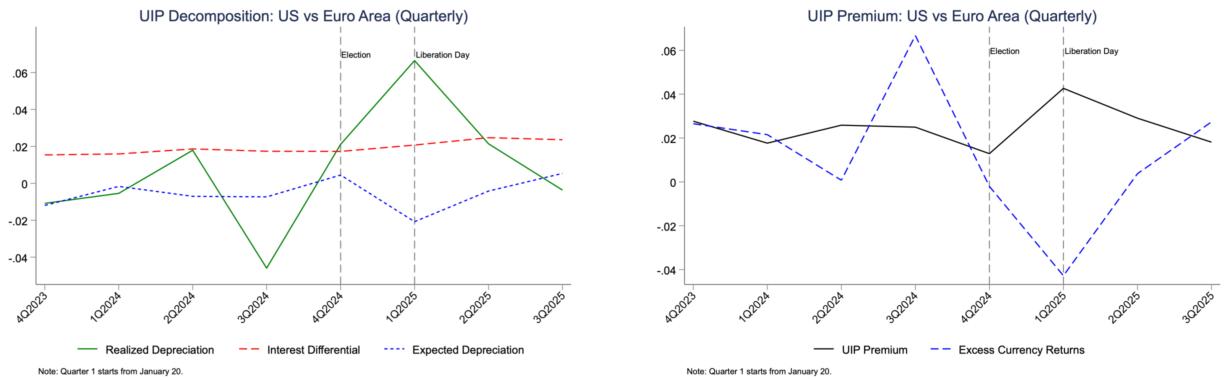
Note: Table A.2 reports the average U.S. tariff rates between January 1, 2025 and October 1, 2025. We downloaded the tariff data from WTO and IMF (2025) as of October 17, 2025, at the HS-6 level of classification. To compute the average U.S. tariff rate, we use 2024 import weights to aggregate tariff rates across products.

Figure A.4. Tariff Threats - not implemented and future implementation uncertain



Note: Figure A.4 visualizes tariff threats between January 20, 2025 and December 24, 2025. The data for the tariff threats, implementations, and planned implementations were compiled from three main sources. The core of the data is from the Trade Compliance Resource Hub Trump 2.0 Tariff Tracker (`https://www.tradecomplianceresourcehub.com/2025/06/27/trump-2-0-tariff-tracker/#updates`). It presents a list from Reed Smith's International Trade and National Security team that tracks the latest threatened and implemented U.S. tariffs.

Figure A.5. Decomposing the UIP Condition and Excess Currency Returns



(a) Interest Differential and Depreciation **(b)** UIP and Excess Currency Returns

Note: *Figure A.5a plots the components, whereas Figure A.5b plots the UIP premium calculated with Consensus survey data and realized excess currency returns. All calculations are for dollar vs euro. We use the convention of Kalemli-Özcan (2019) and Kalemli-Özcan and Varela (2021) while constructing UIP premiums, where the latter paper also links the UIP premium to policy uncertainty. To capture the timeline of tariff policy changes, in this figure and throughout the paper, we calculate quarterly figures with the first quarter of 2025 starting on January 20, 2025 with the inauguration and move each quarter by 20 days accordingly. With this definition of quarters, we are able to capture both Liberation Day announcements and the major tariff rate increases in February and March 2025 in the same quarter.*

B Model Appendix

Our model is a stylized two-country version of KSY, whereby the supply side of the economy is simplified to an endowment economy under flexible prices. It corresponds to the version of the model in Section 4. To that baseline, we add risk-sensitive households (via CARA utility) and risk-sensitive financial intermediaries (via a mean-variance optimization problem). This document is intended to serve as a standalone derivation of the stylized model.

The model leads to higher precautionary savings and UIP premium. Both of these, in turn, lead to a decline in demand for the dollar, which is consistent with the behavior observed around the Liberation Day announcement, as documented by Jiang et al. (2025). An alternative mechanism is proposed by Itskhoki and Mukhin (2025), who argue that when the tariff-imposing country has a negative net foreign asset position (as in the case of the United States) and its liabilities are denominated in domestic currency while its assets are denominated in foreign currency, an improvement in the trade balance can require a depreciation of the domestic currency. Such a depreciation reduces the real value of liabilities while increasing the value of foreign assets.

B.1 Set-up

Timing and countries. Home (subscript H) and Foreign (subscript F) have representative households. Each country is endowed with its own tradable good, $y_{H,t}$ and $y_{F,t}$. One-period nominal discount bonds in each currency are traded internationally.

Preferences and intratemporal aggregator. Each household consumes a CES bundle of Home and Foreign Goods:

$$\max_{\{C_{H,t},c_{H,H,t},c_{H,F,t}\}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t\, u(C_{H,t})\right], \qquad C_{H,t} = \left[(1-\gamma_H)\, c_{H,H,t}^{\frac{\theta-1}{\theta}} + \gamma_H\, c_{H,F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \;\; \theta > 0,\; \theta \neq 1.$$

(Home bias parameter $1-\gamma_H \in (0,1)$; Foreign reliance on Home is $1-\gamma_F$.)

Given home-currency good prices $p_{H,H,t} = p_{H,t}, p_{H,F,t}$, the unit-expenditure (CPI) index and Hicksian demands are

$$P_{H,t} = \Big[(1-\gamma_H)\, p_{H,t}^{1-\theta} + \gamma_H\, p_{H,F,t}^{1-\theta}\Big]^{\frac{1}{1-\theta}}, \quad c_{H,H,t} = (1-\gamma_H)\Big(\tfrac{p_{H,t}}{P_{H,t}}\Big)^{-\theta} C_{H,t}, \quad c_{H,F,t} = \gamma_H \Big(\tfrac{p_{H,F,t}}{P_{H,t}}\Big)^{-\theta} C_{H,t}.$$

Foreign utility maximization, CPI $P_{F,t}$ and demands $c_{F,H,t}, c_{F,F,t}$ are defined analogously with prices in Foreign currency $p_{F,H,t}, p_{F,t}$.

Law of One Price, tariffs and exchange rates. Let $\mathcal{E}_t$ be the nominal exchange rate (Home currency per unit of Foreign currency). For each good,

$$p_{F,H,t} = \frac{p_{H,t}}{\mathcal{E}_t} \qquad p_{H,F,t} = \mathcal{E}_t\, p_{F,t} \tau_t$$

where τ_t is a gross import tariff rate imposed by the home country on the foreign country.

The gross tariff rate is a random variable, whereby $\tau_t = e^{\hat{\tau}_t}$ and $\hat{\tau}_t$ is independently and identically distributed following a normal distribution: $\hat{\tau}_t \sim \mathcal{N}(0, \sigma_{t-1}^2)$. We allow for the variance of $\hat{\tau}_t$ to exogenously vary across time. Notably, we choose the following timing convention. The next period's tariff shock depends on a variance term that is known today: $\text{Var}_t(\hat{\tau}_{t+1}) = \sigma_t^2$. We do this for notational ease in capturing uncertainty. In our stylized model, to capture uncertainty, we will consider one-time changes in σ_t^2, which will impact next period's state variable, $\hat{\tau}_t$. That is today uncertainty increases about tomorrow's tariffs.

Home and foreign budget constraints with nominal bonds.

$$P_{H,t}C_{H,t} + V_{H,t-1} = p_{H,t}y_{H,t} + \frac{V_{H,t}}{R_{H,t}} + T_t$$
$$P_{F,t}C_{F,t} + V_{F,t-1} = p_{F,t}y_{F,t} + \frac{V_{F,t}}{R_{F,t}}$$

where $V_{H,t}$ and $V_{F,t}$ are nominal bonds denominated in local currency and T_t are lump-sum transfers to rebate households for tariff revenues.

Policy. Let us assume monetary policy in the two countries, perfectly stabilizes the aggregate price level such that $P_{H,t} = P_{F,t} = 1 \; \forall t$.

B.2 Households' Problem

The Home household maximizes expected discounted utility over an infinite horizon subject to a sequence of nominal budget constraints:

$$\begin{aligned} \max_{\{C_{H,t}, V_{H,t}\}_{t=0}^{\infty}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_H^t u(C_{H,t}) \\ \text{s.t.} \quad P_{H,t}C_{H,t} + V_{H,t-1} &= p_{H,t}y_{H,t} + \frac{V_{H,t}}{R_{H,t}} + T_t \end{aligned}$$

for all $t \geq 0$, given initial debt $V_{H,-1}$.

The Lagrangian is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_H^t \Big[u(C_{H,t}) + \lambda_{H,t} \Big(p_{H,t}y_{H,t} - P_{H,t}C_{H,t} - V_{H,t-1} + \tfrac{V_{H,t}}{R_{H,t}} \Big) \Big].$$

FOCs with respect to $C_{H,t}$ and $V_{H,t}$:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_{H,t}} : \quad & u'(C_{H,t}) - \lambda_{H,t} P_{H,t} = 0, \\ \frac{\partial \mathcal{L}}{\partial V_{H,t}} : \quad & \frac{\lambda_{H,t}}{R_{H,t}} - \beta_H \mathbb{E}_t[\lambda_{H,t+1}] = 0. \end{aligned}$$

Since policy perfectly stabilizes the aggregate price level we obtain the following Euler equation:

$$u'(C_{H,t}) = \beta_H R_{H,t} \, \mathbb{E}_t[u'(C_{H,t+1})] \, .$$

Using the CARA form, $u'(c) = e^{-\alpha c}$,

$$e^{-\alpha C_{H,t}} = \beta_H R_{H,t}\,\mathbb{E}_t\left[e^{-\alpha C_{H,t+1}}\right].$$

Since uncertainty is driven by tariff shocks, in line with our second-order approximation detailed below, $C_{H,t+1}$ is approximately conditionally normal,

$$C_{H,t+1} \mid \mathcal{F}_t \approx \mathcal{N}(\mathbb{E}_t C_{H,t+1},\ \mathrm{Var}_t(C_{H,t+1})).$$

Then

$$\mathbb{E}_t\left[e^{-\alpha C_{H,t+1}}\right] = \exp\left(-\alpha\,\mathbb{E}_t C_{H,t+1} + \tfrac{1}{2}\alpha^2 \mathrm{Var}_t(C_{H,t+1})\right).$$

So the Euler equation is:

$$\begin{aligned} e^{-\alpha C_{H,t}} &= \beta_H R_{H,t} e^{\left(-\alpha\,\mathbb{E}_t C_{H,t+1} + \frac{1}{2}\alpha^2 \mathrm{Var}_t(C_{H,t+1})\right)} \\ e^{\left(\alpha(\mathbb{E}_t C_{H,t+1} - C_{H,t}) - \frac{1}{2}\alpha^2 \mathrm{Var}_t(C_{H,t+1})\right)} &= \beta_H R_{H,t}. \end{aligned}$$

Taking logs:

$$\alpha(\mathbb{E}_t C_{H,t+1} - C_{H,t}) = \ln(\beta_H R_{H,t}) + \tfrac{1}{2}\alpha^2 \mathrm{Var}_t(C_{H,t+1}).$$

We shall assume that the foreign household, has CRRA utility instead of CARA utility. The foreign household's problem then yields:

$$1 = \beta_F R_{F,t}\,\mathbb{E}_t\left[\left(\frac{C_{F,t+1}}{C_{F,t}}\right)^{-\alpha}\right]$$

B.3 Relative Demand Conditions

Standard CES structure yields the following expressions:

$$\frac{c_{H,H,t}}{c_{H,F,t}} = \frac{1-\gamma_H}{\gamma_H}\left(\frac{p_{H,t}}{p_{H,F,t}}\right)^{-\theta}$$
$$\frac{c_{F,H,t}}{c_{F,F,t}} = \frac{\gamma_F}{1-\gamma_F}\left(\frac{p_{F,H,t}}{p_{F,t}}\right)^{-\theta}$$

B.4 Intermediaries' Problem

Redefine bond variable inclusive of interest payments. Intermediaries solve a mean-variance optimization problem:

$$\max_{V_{H,t},V_{F,t}} \quad \mathbb{E}_t \pi_{H,t+1} - \frac{\chi_t}{2}\,\mathrm{Var}_t(\pi_{H,t+1})$$

subject to the resource constraint at t,

$$0 = \mathcal{E}_t \frac{V_{F,t}}{R_{F,t}} + \frac{V_{H,t}}{R_{H,t}},$$

and with profits (in Home currency) realized at $t+1$,

$$\pi_{H,t+1} = V_{H,t} + \mathcal{E}_{t+1} V_{F,t}.$$

Using the constraint to substitute out $V_{H,t} = -R_{H,t}\mathcal{E}_t \frac{V_{F,t}}{R_{F,t}}$, we obtain

$$\pi_{H,t+1} = V_{F,t}\left(\mathcal{E}_{t+1} - \mathcal{E}_t \frac{R_{H,t}}{R_{F,t}}\right) \tag{B.1}$$

Therefore, the Lagrangian can be written as

$$\mathcal{L}_t = \mathbb{E}_t\Big[V_{F,t}\Big(\mathcal{E}_{t+1} - \mathcal{E}_t\frac{R_{H,t}}{R_{F,t}}\Big)\Big] - \frac{\chi_t}{2}(V_{F,t})^2\,\mathrm{Var}_t\Big(\mathcal{E}_{t+1}\Big),$$

where $\chi_t > 0$ is an aversion to risk. First order condition (interior) yields:

$$0 = \mathbb{E}_t\Big[\Big(\mathcal{E}_{t+1} - \mathcal{E}_t\frac{R_{H,t}}{R_{F,t}}\Big)\Big] - \chi_t(V_{F,t})\,\mathrm{Var}_t(\mathcal{E}_{t+1}), \tag{B.2}$$

$$\iff \quad \mathcal{E}_t\frac{R_{H,t}}{R_{F,t}} = \mathbb{E}_t[\mathcal{E}_{t+1}] - \chi_t(V_{F,t})\,\mathrm{Var}_t(\mathcal{E}_{t+1}) \tag{B.3}$$

Dividing both sides by $\mathcal{E}_t$:

$$\frac{R_{H,t}}{R_{F,t}} = \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} - \chi_t\frac{V_{F,t}}{\mathcal{E}_t}\,\mathrm{Var}_t(\mathcal{E}_{t+1})$$

Rearranging:

$$\frac{R_{H,t}}{R_{F,t}} = \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} - \chi_t \underbrace{V_{F,t}\mathcal{E}_t}_{Quantity\ of\ Risk} \underbrace{\mathrm{Var}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)}_{Price\ of\ Risk}$$

$$\frac{R_{H,t}}{R_{F,t}} = \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} + \chi_t V_{H,t}\frac{R_{F,t}}{R_{H,t}}\mathrm{Var}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)$$

This yields a UIP condition with a deviation based on the quantity of risk (based on the open position of intermediaries) multiplied by the price of risk (variance of depreciation). In the usual interpretation, with $R_{H,t} > R_{F,t}$, $V_{F,t} < 0$ as intermediaries borrow in foreign currency (short foreign bonds) and lend in home currency (long home bonds), so as variance increases the UIP wedge increases.

Let us suppose risk aversion is such that $\chi_t = \chi\frac{R_{H,t}}{R_{F,t}}\frac{1}{V_{H,t}}$. For the stylized model, this allows us to simplify the last equation above into an expression with a convenient risk premium expression, ρ_t, which is not sensitive to the quantity of risk and instead is only sensitive to the price of risk:

$$\frac{R_{H,t}}{R_{F,t}} = \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} + \underbrace{\chi \text{Var}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)}_{\rho_t}$$

We assume that the intermediation profits are transferred to the foreign household.[14]

B.5 Equilibrium Definition

An equilibrium comprises 8 sequences $\{p_{H,t}, p_{F,t}, \mathcal{E}_t, R_{H,t}, R_{F,t}, V_{H,t}, C_{H,t}, C_{F,t}\}_{t=0}^{\infty}$ such that for a given sequence of exogenous variables $\{\tau_t, y_{H,t}, y_{F,t}\}_{t=0}^{\infty}$, equations (B.4)-(B.11) hold:

- Euler equations:

$$\alpha(\mathbb{E}_t C_{H,t+1} - C_{H,t}) = \ln(\beta_H R_{H,t}) + \frac{1}{2}\alpha^2 \text{Var}_t(C_{H,t+1}) \tag{B.4}$$

$$1 = \beta_F R_{F,t}\, \mathbb{E}_t\left[\left(\frac{C_{F,t+1}}{C_{F,t}}\right)^{-\alpha}\right] \tag{B.5}$$

- CPI equations with price level substituted out:

$$1 = \left[(1-\gamma_H)\, p_{H,t}^{1-\theta} + \gamma_H \left(\mathcal{E}_t\, p_{F,t}\tau_t\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{B.6}$$

$$1 = \left[(1-\gamma_F)\, {p_{F,t}}^{1-\theta} + \gamma_F \left(\frac{p_{H,t}}{\mathcal{E}_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{B.7}$$

- UIP condition:

$$\frac{R_{H,t}}{R_{F,t}} = \frac{\mathbb{E}_t[\mathcal{E}_{t+1}]}{\mathcal{E}_t} + \chi \text{Var}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) \tag{B.8}$$

[14] We make this simplification for analytical simplicity and quantitatively this does not make a significant difference as noted in Kekre and Lenel (2024).

- Goods market clears for each country in both periods

$$y_{H,t} = (1-\gamma_H)\left(\frac{p_{H,t}}{1}\right)^{-\theta} C_{H,t} + \gamma_F \left(\frac{p_{H,t}}{\mathcal{E}_t}\right)^{-\theta} C_{F,t} \tag{B.9}$$

$$y_{F,t} = \gamma_H \left(\frac{\mathcal{E}_t\, p_{F,t}\tau_t}{1}\right)^{-\theta} C_{H,t} + (1-\gamma_F)\left(\frac{p_{F,t}}{1}\right)^{-\theta} C_{F,t} \tag{B.10}$$

- Balance of payments equation

$$\begin{aligned}
\frac{V_{H,t}}{R_{H,t}} &= V_{H,t-1} - \mathrm{NX}_t \\
&= V_{H,t-1} - p_{H,t} c_{F,H,t} + p_{F,t}\mathcal{E}_t c_{H,F,t} \\
&= V_{H,t-1} - p_{H,t}\gamma_F \left(\frac{p_{H,t}}{\mathcal{E}_t}\right)^{-\theta} C_{F,t} + p_{F,t}\mathcal{E}_t\gamma_F \left(\frac{\mathcal{E}_t\, p_{F,t}\tau_t}{1}\right)^{-\theta} C_{H,t}
\end{aligned} \tag{B.11}$$

B.6 2nd Order Approximation

Let us assume that the endowment is fixed and let us use the Cole and Obstfeld (1991) parametrization $\alpha = \theta = 1$.[15] When the two countries are different in size, so even if home bias parameters are the same trade can be unbalanced at the steady state. To achieve this, in the general derivation, we normalize home country's consumption $C_H = 1$ and let C_F be some constant (i.e., the rest of the world can be larger). Later we will assume symmetry for the stylized model.

We conduct a second-order approximation around a steady state with unconditional moments evaluated at the ergodic distribution. Hat variables denote deviation from this steady state:

[15]When $\theta = 1$, the consumption price basket becomes a Cobb-Douglas function that is exactly log-linear.

B.6.1 Steady State

- Euler equations:

$$\ln(\beta R_H) = -\frac{1}{2}\sigma_c^2 \rightarrow R_H = \beta_H^{-1} e^{-\frac{1}{2}\sigma_c^2}$$
$$R_F = \beta_F^{-1}$$

We'll assume in our work that volatility is small enough that the steady-state gross interest rate is greater than 1: $R_H = \beta_H^{-1} e^{-\frac{1}{2}\sigma_c^2} > 1$.

- All prices and exchange rate will be 1 at the steady state:

$$P_H = P_F = \mathcal{E} = 1$$

- UIP condition:

$$\frac{R_H}{R_F} = 1 + \chi \sigma_{\mathcal{E}}^2$$

Plugging in what we know from the Euler equation we can show how the endogenous volatility of the exchange rate is related to the endogenous volatility of consumption:

$$\frac{R_H}{R_F} = 1 + \chi \sigma_{\mathcal{E}}^2 = \frac{\beta_H^{-1} e^{-\frac{1}{2}\sigma_c^2}}{\beta_F^{-1}}$$

Importantly, σ_c^2 and $\sigma_{\mathcal{E}}^2$ are a function of the volatility of tariffs, σ^2, at the steady state. For the existence of a steady state, with a constant exchange rate, we assume that β_H and β_F are such that the equality above holds.

- Goods market clears for each country in both periods

$$y_H = 1 + \underbrace{\gamma_H(C_F - 1)}_{\bar{\text{NX}}} = 1 + \bar{\text{NX}}$$

$$y_F = 1 - \gamma_H(C_F - 1) = 1 - \bar{\text{NX}}$$

- Balance of payments equation

$$\begin{aligned}
\frac{V_H}{R_H} &= V_H - \gamma_H C_{F,t} + \gamma_H \\
&= V_H - \underbrace{\gamma_H(C_{F,t} - 1)}_{\bar{\text{NX}}} \\
\frac{V_H}{R_H} &= V_H - \bar{\text{NX}} \\
V_H &= \frac{R_H}{R_H - 1}\bar{\text{NX}}
\end{aligned}$$

These expressions demonstrate that setting the relative size of the countries with C_F is equivalent to setting net exports, which helps determine size of steady-state endowment and debt. If two countries both allocate 10% of their consumption to foreign goods, but one of them is twice the size of the other one, the larger country will be a net importer.

B.6.2 Approximation

Given the parametrization, with $\alpha = \theta = 1$, the model is already highly linear except for the variance terms, so we shall focus on those. [16]

There are two key variances in the model $\text{Var}_t(C_{t+1})$ and $\text{Var}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)$. Any variable can be expressed as the steady-state value multiplied by percent deviation: $X_t = X(1 + \hat{X}_t)$.

[16] We assume near equivalence of log deviation and percent deviation from the steady state. Additionally with consumption and prices normalized to 1, many terms can be converted into hat variables by taking logs.

Then we can write:

$$\begin{aligned}\text{Var}(C_{t+1}) &= \text{Var}(C(1+\hat{C}_{t+1})) \\ &= \text{Var}(\hat{C}_{t+1})\end{aligned}$$

Similarly, since $log\left(\frac{1+\hat{\mathcal{E}}_{t+1}}{1+\hat{\mathcal{E}}_t}\right) \approx \hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t$ and $e^x \approx 1+x$, so we have:

$$\begin{aligned}\text{Var}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) &= \text{Var}_t\left(\frac{1+\hat{\mathcal{E}}_{t+1}}{1+\hat{\mathcal{E}}_t}\right) \\ &= \text{Var}_t\left(e^{log\left(\frac{1+\hat{\mathcal{E}}_{t+1}}{1+\hat{\mathcal{E}}_t}\right)}\right) \\ &\approx \text{Var}_t\left(1+\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t\right) \\ &= \text{Var}_t\left(\hat{\mathcal{E}}_{t+1}\right)\end{aligned}$$

With these the Euler equation and the UIP condition read as follows:

$$\begin{aligned}(\mathbb{E}_t\hat{C}_{H,t+1} - \hat{C}_{H,t}) &= \hat{i}_{H,t} + \frac{1}{2}\text{Var}(\hat{C}_{t+1}) \\ \hat{i}_{H,t} - \hat{i}_{F,t} &= \mathbb{E}_t[\hat{\mathcal{E}}_{t+1}] - \hat{\mathcal{E}}_t + \chi\text{Var}_t\left(\hat{\mathcal{E}}_{t+1}\right)\end{aligned}$$

We have a system with three state variables: $\hat{\tau}_t, \hat{V}_{t-1}, \sigma_t^2$. With a second order approximation, every endogenous variable, $\hat{x}_t$ (e.g., including $\hat{\mathcal{E}}_{t+1}$ and $\hat{C}_{t+1}$) will be some linear and quadratic function of the state variables:

$$\begin{aligned}\hat{x}_t = a_1\hat{\tau}_t + a_2\hat{V}_{H,t-1} + a_3\sigma_t^2 + a_4\hat{\tau}_t^2 + a_5\hat{V}_{H,t-1}^2 + a_6(\sigma_t^2)^2 \\ + a_7\hat{\tau}_t\hat{V}_{H,t-1} + a_8\hat{\tau}_t\sigma_t^2 + a_9\hat{V}_{H,t-1}\sigma_t^2.\end{aligned}$$

Iterating one period forward:

$$\hat{x}_{t+1} = a_1\hat{\tau}_{t+1} + a_2\hat{V}_{H,t} + a_3\sigma^2_{t+1} + a_4\hat{\tau}^2_{t+1} + a_5\hat{V}^2_{H,t} + a_6(\sigma^2_{t+1})^2$$
$$+ a_7\hat{\tau}_{t+1}\hat{V}_{H,t} + a_8\hat{\tau}_{t+1}\sigma^2_{t+1} + a_9\hat{V}_{H,t}\sigma^2_{t+1}.$$

Let us assume that we are interested in one-time increases in uncertainty, so $\sigma_{t+j} = 0 \forall j > 0$ and this is known by agents. If the variance of $\hat{\tau}_{t+1}$ is σ_t^2, then

$$\begin{aligned}
\text{Var}_t(\hat{x}_{t+1}) &= \text{Var}_t\Big(a_1\hat{\tau}_{t+1} + a_4\hat{\tau}^2_{t+1} + a_7\hat{\tau}_{t+1}\hat{V}_{H,t} + a_8\hat{\tau}_{t+1}\sigma^2_{t+1}\Big) \\
&= \text{Var}_t\Big(\big(a_1 + a_7\hat{V}_{H,t}\big)\hat{\tau}_{t+1} + a_4\hat{\tau}^2_{t+1}\Big) \\
&= \big(a_1 + a_7\hat{V}_{H,t}\big)^2\sigma_t^2 \ + \ 2a_4^2\sigma_t^4
\end{aligned}$$

We shall assume $\sigma_t < 1$. Given this, for small shocks, higher order terms will be near zero (i.e. $\big(a_7\hat{V}_{H,t}\big)^2\sigma_t^2 \ + \ 2a_4^2\sigma_t^4 \approx 0$. That is we choose to focus on a variance that is only a function of the exogenous variance of $\hat{\tau}_t$, and simplify away the endogenous time-varying component.[17] With that, the variance term will be directly a function of the dependence of the endogenous variable on $\hat{\tau}_t$: that is we have $\text{Var}_t(\hat{x}_{t+1}) \approx a_1^2\sigma_t^2$. With that we can define the approximated equilibrium as follows:

An approximated equilibrium comprises 8 sequences $\{\hat{p}_{H,t}, \hat{p}_{F,t}, \hat{\mathcal{E}}_t, \hat{i}_{H,t}, \hat{i}_{F,t}, \hat{V}_{H,t}, \hat{C}_{H,t}, \hat{C}_{F,t}\}_{t=0}^{\infty}$ such that, given exogenous variables $\{\hat{\tau}_t, \sigma_t^2\}_{t=0}^{\infty}$, the following equations hold:

- Euler equations:

$$(\mathbb{E}_t\hat{C}_{H,t+1} - \hat{C}_{H,t}) = \hat{i}_{H,t} + \eta\sigma_t^2 \tag{B.12}$$

$$(\mathbb{E}_t\hat{C}_{F,t+1} - \hat{C}_{F,t}) = \hat{i}_{F,t} \tag{B.13}$$

[17] That is via the $a_7\hat{V}_{H,t}$ term, there could otherwise be endogenous time variation in how the variance of $\hat{C}_{t+1}$ and $\hat{\mathcal{E}}_{t+1}$ depends on the underlying variance of tariffs. We assume that in our model and context of shocks, this is small enough to simplify away.

- CPI equations with price level substituted out:

$$0 = (1-\gamma_H)\,\hat{p}_{H,t} + \gamma_H\,(\hat{\mathcal{E}}_t + \hat{p}_{F,t} + \hat{\tau}_t) \tag{B.14}$$

$$0 = (1-\gamma_F)\,\hat{p}_{F,t} + \gamma_F(\hat{p}_{H,t} - \hat{\mathcal{E}}_t) \tag{B.15}$$

- UIP condition with a wedge that endogenously widens as outstanding debt increases and as volatility increases:

$$\hat{i}_{H,t} - \hat{i}_{F,t} = \mathbb{E}_t\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t + \kappa\sigma_t^2 \tag{B.16}$$

- Goods market clears for each country in both periods

$$0 = (1-\gamma_H)\left(\hat{C}_{H,t} - \hat{p}_{H,t}\right) + \gamma_H C_F\left(\hat{C}_{F,t} + \hat{\mathcal{E}}_t - \hat{p}_{H,t}\right) \tag{B.17}$$

$$0 = \gamma_F\left(\hat{C}_{H,t} - \hat{\mathcal{E}}_t - \hat{p}_{F,t} - \hat{\tau}_t\right) + (1-\gamma_F)C_F\left(\hat{C}_{F,t} - \hat{p}_{F,t}\right) \tag{B.18}$$

- Balance of payments equation with $\hat{V}_{H,t}$ as net debt of the home country:[18]

$$\hat{V}_{H,t} = R_H\hat{V}_{H,t-1} - \Xi\left[C_F(\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - (\hat{C}_{H,t} - \hat{\tau}_t)\right] + \hat{i}_{H,t} \tag{B.23}$$

For the analytical simplicity of the stylized model, let us additionally assume symmetry (i.e.

[18] The derivation is as follows:

$$\frac{V_{H,t}}{R_{H,t}} = V_{t-1} - p_{H,t}\gamma_F\left(\frac{p_{H,t}}{\mathcal{E}_t}\right)^{-\theta} C_{F,t} + p_{F,t}\mathcal{E}_t\gamma_H\left(\frac{\mathcal{E}_t\,p_{F,t}\tau_t}{1}\right)^{-\theta} C_{H,t} \tag{B.19}$$

$$\frac{V_H}{R_H}(\hat{V}_{H,t} - \hat{i}_{H,t}) = V_H\hat{V}_{H,t-1} - \left(\gamma_H C_F(\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - \gamma_H(\hat{C}_{H,t} - \hat{\tau}_t)\right) \tag{B.20}$$

$$(\hat{V}_{H,t} - \hat{i}_{H,t}) = R_H\hat{V}_{H,t-1} - \frac{R_H}{V_H}\left(\gamma_H C_F(\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - \gamma_H(\hat{C}_{H,t} - \hat{\tau}_t)\right) \tag{B.21}$$

$$\hat{V}_{H,t} = R_H\hat{V}_{H,t-1} - \underbrace{\frac{R_H - 1}{\bar{\mathrm{NX}}}\gamma_H}_{\Xi}\left(C_F(\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - (\hat{C}_{H,t} - \hat{\tau}_t)\right) + \hat{i}_{H,t} \tag{B.22}$$

where the last two lines follow from $V_H = \frac{R_H}{R_H - 1}\bar{\mathrm{NX}}$.

setting $\gamma_H = \gamma_F = \gamma$) and consequently an initial position of zero debt. Then equilibrium conditions read as follows:

An approximated equilibrium comprises 8 sequences $\{\hat{p}_{H,t}, \hat{p}_{F,t}, \hat{\mathcal{E}}_t, \hat{i}_{H,t}, \hat{i}_{F,t}, \hat{V}_{H,t}, \hat{C}_{H,t}, \hat{C}_{F,t}\}_{t=0}^{\infty}$ such that, given exogenous variables $\{\hat{\tau}_t, \sigma_t^2\}_{t=0}^{\infty}$, the following equations hold:

- Euler equations:

$$(\mathbb{E}_t \hat{C}_{H,t+1} - \hat{C}_{H,t}) = \hat{i}_{H,t} + \eta \sigma_t^2 \tag{B.24}$$

$$(\mathbb{E}_t \hat{C}_{F,t+1} - \hat{C}_{F,t}) = \hat{i}_{F,t} \tag{B.25}$$

- CPI equations with price level substituted out:

$$0 = (1-\gamma)\,\hat{p}_{H,t} + \gamma\,(\hat{\mathcal{E}}_t + \hat{p}_{F,t} + \hat{\tau}_t) \tag{B.26}$$

$$0 = (1-\gamma)\,\hat{p}_{F,t} + \gamma(\hat{p}_{H,t} - \hat{\mathcal{E}}_t) \tag{B.27}$$

- UIP condition with a wedge that endogenously widens as volatility increases:

$$\hat{i}_{H,t} - \hat{i}_{F,t} = \mathbb{E}_t \hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t + \kappa \sigma_t^2 \tag{B.28}$$

- Goods market clears for each country in both periods

$$0 = (1-\gamma)\left(\hat{C}_{H,t} - \hat{p}_{H,t}\right) + \gamma\left(\hat{C}_{F,t} + \hat{\mathcal{E}}_t - \hat{p}_{H,t}\right) \tag{B.29}$$

$$0 = \gamma\left(\hat{C}_{H,t} - \hat{\mathcal{E}}_t - \hat{p}_{F,t} - \hat{\tau}_t\right) + (1-\gamma)\left(\hat{C}_{F,t} - \hat{p}_{F,t}\right) \tag{B.30}$$

- Balance of payments equation with $\hat{V}_{H,t}$ as net debt of the home country:

$$R_H^{-1} \hat{V}_{H,t} = \hat{V}_{H,t-1} - \gamma\left[(\hat{C}_{F,t} + \hat{\mathcal{E}}_t) - (\hat{C}_{H,t} - \hat{\tau}_t)\right] \tag{B.31}$$

C Data and Code Availability

All data and code are available upon request.

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